Notre Dame Journal of Formal Logic
Volume 27, Number 4, October 1986

Strong Normalization for Typed Terms with Surjective Pairing

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1 Introduction In this note we describe a simple method for reducing strong normalization (henceforth to be abbreviated as SN) for PRIM($\mathfrak{T}$), the primitive recursive functionals with surjective pairing, to SN for PRIM($\mathfrak{T}_f$), the primitive recursive functionals with functional types only. The set of contraction rules includes the "surjectivity contraction", that is to say the contraction of a redex of the form $\boldsymbol{p}(\boldsymbol{p}_1 t)(\boldsymbol{p}_2 t)$ to t, for all t of the appropriate type, where $\boldsymbol{p}$ is a pairing operator with inverses $\boldsymbol{p}_1$, $\boldsymbol{p}_2$.

SN for PRIM($\mathfrak{T}$) almost automatically entails SN for certain related systems of typed terms and conversion rules, via suitable homomorphic embeddings into PRIM($\mathfrak{T}$) which preserve nontrivial reductions (t reduces trivially to t' iff t' and t are identical modulo renaming bound variables). Examples of such systems are: the typed $\lambda\eta$-calculus with surjective pairing; (terms describing) natural deductions in intuitionistic predicate logic, with respect to proper conversions (detour conversions) but without permutation conversions (cf. [10], 4.1.3); the terms in the first-order fragment $\mathbf{ML}_0$ of Martin-Löf's theory of types ([6]; $\mathbf{ML}_0$ is described for example in [2]). In the case of the first two examples our method for reducing SN for arbitrary terms to SN for the terms without $\times$ in their types can also be applied directly.

In the literature there are several proofs of SN for either PRIM($\mathfrak{T}$) itself [3] or for natural deduction systems containing at least the rules for $\rightarrow$, $\wedge$, $\forall$ and induction, which technically are very close to PRIM($\mathfrak{T}$), so that these proofs can be adapted to PRIM($\mathfrak{T}$) (e.g., [8], [4], [5]).

None of these published proofs consider the (analogue of) surjective pairing contraction SP. At least for Gandy's proof it is easy to show that SP is covered as well.

A straightforward extension of the proof in [10], 2.2, based on Tait's notion [9] of computability, is also possible, as was shown by R. de Vrijer (1982, unpublished). The result and method of Bercovici [1] are similar and can presumably be extended to PRIM($\mathfrak{T}$). De Vrijer adapted the definition of strong computability SC in [10], 2.2.13, as follows:

Received October 17, 1984; revised March 12, 1985