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# Modality and Possibility in Some Intuitionistic Modal Logics

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***1 Introduction*** Traditionally, since the time of Aristotle, modal logic was built upon two main concepts, namely those of necessity and possibility, currently taken in an ontological sense. In the formal language they are represented by two unary operators, $L$ for necessity and $M$ for possibility. In classical logic, these operators are considered to be dual to each other and mutually definable through the formulas $M \leftrightarrow \neg L \neg$ and $L \leftrightarrow \neg M \neg$. However if we work on an intuitionistic nonmodal base logic, then some properties of the negation are weakened, the duality disappears, and it is commonly admitted that both equivalences cannot remain valid, because they lead to conclusions stronger than wished (see [4]). Of course one could ignore one of the two modal operators, but we think this pointless, because the dual interpretation of one of them gives natural birth to the other one.[1] On the other hand, several studies of intuitionistic modal logic have been published where neither of the two equivalences holds, the operators $L$ and $M$ being both primitive and independent, and linked through other indirect properties; see [19], [4]–[6], [9], [10], [18], [17] and the global studies of [20], [3], and [8].

Our choice is to try to apply Gödel's proposal for S4 (from [13]) to an intui intuitionistic base, that is, to consider $L$ as a primitive symbol with implicative S4-type axioms and to define $M$ as $\neg L \neg$. Here "$p$ is possible" just means that "it is contradictory that $p$ is necessarily contradictory"; we do not start from a philosophical analysis of any concept of possibility (as Aristotle and the Middle-Ages logicians probably did) but rather we make their properties follow from those of a primitive concept of necessity, the link between them being a formula where the "logical" negation plays an important part.[2] So we are formalizing a kind of derived or "negative" concept of possibility and it is in this sense that we would speak of a "logical" possibility rather than of a "philosophical" or "ontological" one. It should be emphasized that the remaining alternative, that of considering $M$ as primitive and defining $L$ as $\neg M \neg$, is not interesting because, even if we adopt very strong axioms for $M$, the simplest properties of $L$ cannot

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