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Finite Kripke Models of **HA** are Locally **PA**

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Introduction In a Kripke model of Heyting's Arithmetic, **HA**, the nodes, when viewed as classical structures, are models of classical arithmetic with (at least) Δ_1^0-induction. In general, it is an open problem which form of induction holds in the classical structures at the nodes of Kripke models. However, in the case of finite Kripke models (i.e., those containing a finite number of nodes) one can show that all these structures satisfy full induction, and consequently are models of full Peano Arithmetic, **PA.** It can also be shown that any Kripke model with an underlying model structure of type ω must contain an infinite number of such Peano models. These results were established in a workshop in Utrecht (1983).

1 Preliminaries Let L be a first-order language with logical constants: $\perp$, $\wedge$, $\vee$, $\rightarrow$, $\forall$, $\exists$, $=$. Let $\neg\phi$ be short for $\phi \rightarrow \perp$ and let $\phi \leftrightarrow \psi$ be short for $(\phi \rightarrow \psi) \wedge (\psi \rightarrow \phi)$. An extension L_D of L is obtained by adding an individual constant $\bar{c}$ for each element c of D. In practice, D shall always be the local domain D_α of some node α in a Kripke model, and we shall write L_α instead of L_{D_α}.

A *Kripke model* $\mathbf{K} = \langle K, \leq, D, I\rangle$ consists of a nonempty set K of nodes, partially ordered by $\leq$, a function D that assigns a nonempty local domain of individuals to each $\alpha \in K$, and a function I that assigns an interpretation function I_α to each $\alpha \in K$. Each I_α assigns values to the individual constants, the function symbols, and the predicate symbols of L_α, so as to provide for a *local model* $\mathbf{M}_\alpha = \langle D_\alpha, I_\alpha\rangle$. The different I_α agree on the values assigned to individual constants that belong to L. Moreover, D and I are to be cumulative in the following sense: if $\alpha \leq \beta$ then $D_\alpha \subseteq D_\beta$, and, for each function symbol or predicate symbol X, $I_\alpha(X) \subseteq I_\beta(X)$. $\mathbf{K}$ is called finite if K is finite.

Since we are interested in a theory with decidable equality it is no restriction to assume that '$=$' is interpreted by the actual identity in each node (cf. [1], p. 184).

Semantic evaluations proceed as usual. We write $\alpha \models \phi$ if ϕ is *true* in the (classical) model $\mathbf{M}_\alpha$, and $\alpha \Vdash \phi$ if α *forces* ϕ. Further, we write $\alpha \Vdash \Gamma$ if for each $\phi \in \Gamma$, $\alpha \Vdash \phi$. The symbol '$\vdash$' shall denote derivability on the strength of intuitionistic logic.

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