

Kripke-Type Semantics for Da Costa's Paraconsistent Logic C_ω

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This paper should be considered as an example of the formal derivation of semantics in connection with propositional logics extending positive logic. These constructions can be applied to many similar problems and should be compared to those proposed in Hacking [4]: The Kripke-type structure of the resulting semantics depends on the previous choice of Gentzen-style formulations. The details of the semantics are read off from the adopted introduction and elimination rules.

In Arruda [1], p. 27, problem 8, the problem is formulated to adapt world semantics to C_ω . The proposed construction of an adequate semantics for C_ω is based on Raggio's Gentzen-type formulation CG_ω of C_ω^* [5], and on the proof-theoretic analysis of intuitionistic logic in Takeuti [6], Section 8.

1 Definition of C_ω and CGP_ω Define C_ω as usual

- (1) $A \supset (B \supset A)$
- (2) $(A \supset B) \supset ((A \supset (B \supset C)) \supset (A \supset C))$
- (3)
$$\frac{A \quad A \supset B}{B}$$
- (4) $A \wedge B \supset C$
- (5) $A \wedge B \supset B$
- (6) $A \supset (B \supset A \wedge B)$
- (7) $A \supset A \vee B$
- (8) $B \supset A \vee B$
- (9) $(A \supset C) \supset ((B \supset C) \supset (A \vee B \supset C))$
- (10) $A \vee \neg A$
- (11) $\neg\neg A \supset A$.

(Cf. Da Costa [2] or Arruda [1].) Let CGP_ω be the restriction of CG_ω to propositional syntax: CGP_ω corresponds to the propositional part of Gentzen's system LK with

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