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Notre Dame Journal of Formal Logic
Volume 27, Number 4, October 1986

Modal Logics with Functional Alternative Relations

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Some families of modal logics form lattices; particularly important examples are the set of extensions of a modal logic and the set of normal extensions of a normal logic. One traditional way of studying such lattices, falling back on previous work in algebra, seeks to establish general properties of very big lattices. Thanks to Kit Fine, Wim Blok, Johan van Benthem, and others, this tradition is very much alive.

But there is also an earlier tradition, related to but perhaps possible to distinguish from the one mentioned, where the ambition is to map out in complete detail sufficiently small lattices. The first work in this vein was Scroggs's celebrated [15], followed by Bull's equally celebrated [1]. Other investigations in the same tradition are exemplified by [2], [6], [18], [19], [21], [22]; and works such as [5], [7], [11] also bear on it. In view of how enormously complicated the big lattices are, this tradition can never hope to develop very far. Nevertheless, where it is viable there may still be some interest in seeing it pursued. In this paper we will offer one such example, exploring the lattice of extensions of the normal modal logic $\boldsymbol{KD_c}$, where the schema D_c. $\Diamond \mathbf{A} \supset \Box \mathbf{A}$ is the converse of the well-known "deontic" schema D. $\Box \mathbf{A} \supset \Diamond \mathbf{A}$. At the outset we may note that the only extensions of $\boldsymbol{KD_c}$ other than the Inconsistent Logic (the normal extension of $\boldsymbol{K}$ by $\bot$) which seem to have been described in the literature are $\boldsymbol{KD!} = \boldsymbol{KDD_c}$ (the smallest normal logic to contain both D and D_c), the Trivial Logic (the normal extension of $\boldsymbol{K}$ by the schema $\Box \mathbf{A} \equiv \mathbf{A}$), and the Verum Logic (the normal extension of $\boldsymbol{K}$ by $\Box \bot$). The relationship between these logics is set out by the chart in Fig. 1. This, then, is the map whose white patches we propose to fill.

*The research reported in this paper was prompted by Brian Chellas's interest in functional modal logics, as explained in [4], and the author would like to acknowledge fruitful exchanges with him, Max Cresswell, and Steve Thomason on this topic, in conversation and in correspondence. The results achieved were presented in the author's invited address to the Australasian Logic Conference at the University of Western Australia in May 1983 and were summarized in the abstract [20]. The author wishes to thank Graham Priest and his colleagues for an exciting and well-organized conference.

Received March 10, 1984