Notre Dame Journal of Formal Logic
Volume 27, Number 4, October 1986

Gentzen Systems, Resolution, and Literal Trees

DANIEL J. DOUGHERTY*

1 Introduction We are concerned with the relationship between various proof systems for propositional logic, with particular emphasis on the size of derivations. Two traditional systems, Gentzen's sequent calculus, and Robinson's resolution method, are investigated by introducing a (somewhat) new formalism, which may be viewed as a common generalization.

Cook and Reckhow, in [4] and [5], studied many logical calculi, including resolution and Gentzen systems, viewing these as nondeterministic algorithms, and reported polynomial time simulation results among certain systems. Both [4] and [5] contain discussions of the connection with computational complexity. More recently, Haken, in his thesis [8], showed that resolution is not a polynomially bounded system. Our emphasis here is on the structure of the derivations themselves, and upon obtaining inference-by-inference transformations, usually preserving the relation of subderivation. Our notation is as follows:

Propositional Logic: We assume an infinite set of literals p_i; these come in complementary pairs, the complement of p is denoted $\bar{p}$. *Formulas* are defined as follows: a literal is a formula, and if $A_1, \ldots, A_k$ are distinct formulas, then $\wedge\{A_1, \ldots, A_k\}$ and $\vee\{A_1, \ldots, A_k\}$ are formulas. We sometimes write $A_1 \wedge \ldots \wedge A_k$ and $A_1 \vee \ldots \vee A_k$. A disjunction of literals $\vee\{p_1, \ldots, p_k\}$ is a *clause*; it is convenient to refer to a clause by juxtaposing its literals: $p_1 p_2 \ldots p_k$.

The *negation* $\sim A$ of a formula A is defined by induction: if A is a literal, then $\sim A$ is $\bar{A}$; if A is $\wedge\{A_1, \ldots, A_k\}$, then $\sim A$ is $\vee\{\sim A_1, \ldots, \sim A_k\}$; if A is $\vee\{A_1, \ldots, A_k\}$, then $\sim$A is $\wedge\{\sim A_1, \ldots, \sim A_k\}$.

All of the systems we consider are refutation systems, that is, they demonstrate unsatisfiability, usually of sets of clauses. We will, however, use the terms proof, refutation, and derivation synonymously to refer to objects in these systems.

Trees: Our trees are finite, rooted trees, which we draw branching downward. If N^- is on the path from the root to N^+, $N^- \neq N^+$, we say N^+ lies *below* N^-, and write $N^- < N^+$. If N^+ lies immediately below N^-, we say that

*This is a portion of the author's Ph.D. dissertation, written under the direction of Prof. James C. Owings, at the University of Maryland. Partial support was provided by NSF Grant MCS-7919418.

Received May 14, 1984; revised March 22, 1985