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Model Constructions in Stationary Logic Part II: Definable Ultrapowers

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In this paper we continue our investigation of techniques for constructing new models of an arbitrary theory T in stationary logic, $L(aa)$. In Part I [2], we discussed the technique of model-theoretic forcing as a tool for building models in $L(aa)$. In this note we present a technique for constructing models by a series of definable ultrapowers.

$L(aa)$ was introduced by Shelah [3] (using the notation $L(Q_{\aleph_1}^{SS})$), and [1] contains the first explicit proofs of completeness, compactness, and omitting types for $L(aa)$. New proofs of these were given in [2] using forcing. (A sketch of the history of $L(aa)$ may be found in Section 8.1 of [1].)

One reason that ultrapowers and ultraproducts have not usually been used in the study of logics with generalized quantifiers like $L(aa)$ and $L(Q_1)$ (logic with the generalized first-order quantifier "there exist uncountably many x") is that if U is a countably incomplete ultrafilter over set I and A is countable, then $\Pi_U A$ will be uncountable. Since the main difficulty in generating models for these logics is in keeping the countable sets from growing and becoming uncountable, the usual ultrapower construction has not been helpful. Definable ultrapowers, originally introduced by Skolem, come to the rescue here. If the set I is countable and only definable functions from I into A are considered, $\pi_U A$ will still be countable. As usual in building definable ultrapowers we will need our language to contain built-in Skolem functions. With this, and by iterating the definable ultrapowers ω_1 times, we will be able to construct standard models of $L(aa)$. In particular we will give a new proof of the compactness theorem for $L(aa)$.[1]

1 Preliminaries We review very briefly the logic $L(aa)$ and the notion of a weak model for $L(aa)$. Our terminology and notation are the same as in [2]. Let $\mathcal{P}_{\omega_1}(A)$ be the set of all countable subsets of A.

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