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On An Implication Connective of RM

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Introduction The Dunn-McCall system *RM* was developed and studied by the "Entailment" school (mainly by Meyer and Dunn), but it can hardly be called "relevance logic" because of theorems like $\sim(A \rightarrow A) \rightarrow (B \rightarrow B)$ and $(A \rightarrow B) \vee (B \rightarrow A)$ (see [1], 29.5); yet it is a strong and decidable logic which still avoids $A \rightarrow (B \rightarrow A)$ and $\sim A \rightarrow (A \rightarrow B)$.

Some new light on *RM* is shed here (so we hope) by investigating an implication connective $\supset$ definable in it by $(A \rightarrow B) \vee B$. "$\supset$" has most of the properties one might expect an implication to have in a paraconsistent logic[1]: respecting M.P., the "official" deduction theorem, and a strong version of the Craig interpolation theorem: $RM \vdash A \supset B$ iff either $RM \vdash B$, or there is an interpolant C for A and B. (In classical logic there is also the possibility that $\vdash \sim A$.) These facts are all proved in Section 1.

In Section 2 we investigate *RM* as a system in the $\{\sim, \vee, \wedge\supset\}$ language. We give a simple axiomatization of its $\{\sim\supset\}$ fragment, which suffices for characterizing the Sugihara matrix.[2] In this fragment $\rightarrow$ is definable (so the Sobociński logic[3] is a proper subsystem of it), but $\vee$ is not. We get the full system *RM* by adjoining some natural axioms concerning $A \vee B$ and $\sim(A \vee B)$ to its $\{\sim\supset\}$ fragment. In contrast to extending with $\vee$ the $\{\sim\rightarrow\}$ fragment, this extension causes no essential changes.

From the simple classical laws concerning combinations of $\sim$ with $\supset$, $\vee$, and $\wedge$, *RM* only lacks $\sim(A \supset B) \supset A$ and $\sim A \supset (A \supset B)$. By adding, in Section 3, the first schema to *RM*, we get a three-valued logic equivalent to what was called RM_3 in [1]. This system might be considered an optimal paraconsistent logic, since its positive fragment (in the $\{\supset, \wedge, \vee\}$ language) is identical with the classical one. It avoids $\sim A \supset (A \supset B)$, but every proper extension of it (closed under substitutions) is equivalent to *PC*.

Preliminaries The system *RM* is obtained from the system *R* by adding to it the mingle axiom $A \rightarrow (A \rightarrow A)$. We assume the reader is acquainted with this system and its properties, as described in [1], 29.3–4.

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