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On Purely Relevant Logics

ARNON AVRON

1 Introduction The system $RMI_{\stackrel{\rightarrow}{\sim}}$ (which consists of the implication-negation axioms of *RM*) was investigated in [3] and shown there to be an optimal relevance logic in its language. We note there, however, that one cannot add to it an *R*-style extensional conjunction $\wedge$, with $A \wedge B \rightarrow A$, $A \wedge B \rightarrow B$ as axioms and the adjunction rule of inference ($A, B \vdash A \wedge B$), without losing its relevance character (see [1], 29.5, and [3], III.8).

This state of affairs is not altogether surprising. Anderson and Belnap faced a similar problem when they came to add to $R_{\stackrel{\rightarrow}{\sim}}$ (or $E_{\stackrel{\rightarrow}{\sim}}$) extensional connectives. In $R_{\stackrel{\rightarrow}{\sim}}$, e.g., the meaning of $\rightarrow$ is given by the "relevant deduction theorem", according to which a sentence of the form $A_1 \rightarrow (A_2 \rightarrow \ldots \rightarrow (A_n \rightarrow B) \ldots)$ is provable in $R_{\stackrel{\rightarrow}{\sim}}$ iff there is a proof in $R_{\stackrel{\rightarrow}{\sim}}$ of B from the assumptions $A_1, \ldots, A_n$ which *uses* all the A_i's. (Here the meaning of "proof" is the usual one, while the meaning of "use" is to be understood according to the relevantist's analysis of this term (see [1], Chapter 1).) Accordingly, if one wishes to add to $R_{\stackrel{\rightarrow}{\sim}}$ an extensional conjunction such that $A \wedge B \vdash A$, $A \wedge B \vdash B$ and $A, B \vdash A \wedge B$ are all valid modes of inference, then he must recognize $A \wedge B \rightarrow A$, $A \wedge B \rightarrow B$ and $A \rightarrow (B \rightarrow A \wedge B)$ as valid sentences. However, it is well known that by adding these schemes to $R_{\stackrel{\rightarrow}{\sim}}$ we get classical logic.

Anderson and Belnap's first step in order to solve this difficulty was to give up $A \rightarrow (B \rightarrow A \wedge B)$ as a valid sentence and to introduce instead adjunction as a new, primitive rule of inference (besides M.P. for $\rightarrow$). A second, unavoidable step was to propose some new concepts of "proof" relative to which some version of the deduction theorem does hold. (In [1] and [5] three competing definitions can be found of what a "proof" in *R* or *E* is.[1] This is an obvious evidence that the relevantists have no clear intuition at this point.) These concepts of proofs all seem ad hoc and entail many absurdities. Consider an example: $A \wedge (B \rightarrow B)$ can be inferred, according to them, if we assume both A and $B \rightarrow B$ but not if we assume A alone, although $B \rightarrow B$ is a logical truth of the system and so it would be ridicuous to pretend assuming it.

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