A New Axiomatization of Belnap's
Conditional Assertion

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1 Introduction   A *conditional assertion* is a statement which succeeds in making an assertion only if the supplied condition of assertion is met. Otherwise, it fails to assert; it is "nonassertive". It is comparable to a conditional bet. Although the notion of asserting something only conditionally has been around for several decades,[1] the definitive characterization is the presentation in [1]. That discussion will be presupposed.

In spite of attempted arguments to the effect that there can be no sense made of conditional assertions ([4], and pp. 338–347 in [5]), there have indeed been successful formalizations of languages with a conditional assertion connective. Most notable among these is Dunn [6].[2] However, the very success of that presentation raises a number of questions. First, there are the initial and explicit philosophical questions about the soundness of the motivation behind the enterprise. In particular, the two-logics structure presented needs scrutinizing. Secondly, there are implicit questions concerning some unfinished business. These are probably best read as challenges to other workers in the field. Finally, there is a serious unraised question about the completeness proofs themselves: there is a point in the proofs which is so susceptible to error that avoiding the pitfall without explicit mention of it could be misconstrued as fortuitous. The axiomatization offered here addresses all of these.

2 Ascertaining assertiveness   Belnap [1] suggests that a formalization of conditional assertions might take a double-barreled approach, first axiomatizing the always-true formulas and then the never-false ones. This is what Dunn [6] does. In fact, he does more in that he also proves that success in one task guarantees success in the other — perhaps thereby proving he has done less. In either case,