

Notes on the Mathematical Aspects of Kripke's Theory of Truth

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1 Introduction Kripke's Theory of Truth [3] is one of the most interesting developments to come along in this area for some time. But the mathematical difficulties may have prevented a wider appreciation of its philosophical virtues. Certainly more is required of the reader than in Tarski's approach; indeed the mathematical machinery involved strictly includes that required by Tarski's theory. But, in fact, the necessary mathematics is simple, elegant, and part of the standard tool-kit of workers in certain areas of mathematical logic and computer science.

What we do in this primarily expository paper is present, in a compact connected fashion, a development of the background mathematics pertinent to Kripke's theory. We largely skip over philosophical motivation; one cannot do better than to read Kripke himself on this. We do not present the body of the theory, only the mathematical skeleton. The paper is self-contained in the mathematical sense, however, with all terms defined and all main results proved.

There are a few items in the treatment here which, although not new, seem to have been ignored by most writers in the area.

It is most common to establish the existence of smallest fixed points by using ordinally indexed sequences of approximations. This is not necessary, and has two distinct drawbacks: First, it is more mathematical paraphernalia than one needs, tending to obscure the inherent simplicity of the subject. Second, it is a construction that only works for special kinds of fixed points, while in Kripke's theory all fixed points have some role to play. Consequently we postpone this technique to the final section; an afterthought rather than a central feature. We do show, however, that there is an analogous "dual" construction of the largest intrinsic fixed point, something that seems to have gone unremarked on in the literature.

Another peculiar feature of this work is its treatment of models. There are none. One is interested in statements, their truth and falsity. So rather than

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