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Solving Functional Equations at Higher Types; Some Examples and Some Theorems

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The solvability of higher type functional equations has been studied by a number of authors. Roughly speaking the literature sorts into four topics: constructive solvability (e.g., Gödel [5], Scott [7]); solvability in all models, i.e., unification (e.g., Andrews [1], Statman [8] and [9]); solvability in models of A.C. (e.g., Church [2], Friedman [4]); and the solvability of special classes of equations (e.g., Scott [7]). In this note we shall consider yet a fifth topic, namely, the solvability of functional equations in extensions of models.

Our main result is the no counterexample theorem. This theorem equates the unsolvability of E in every extension of $\mathfrak{A}$ with the solvability of some other $\tilde{E}$ in $\mathfrak{A}$. The theorem can be iterated and applied to λ theories (in extended languages) as well as to models. Thus, it can be used to explain, in a general way, a phenomenon well illustrated by the case of $\lambda\sqcup$.

$\lambda\sqcup$ is the theory of upper semilattices of monotone functionals. $\lambda\sqcup$ has the property that each of its models can be extended to solve all the fixed point equations

$$Mx = x\ .$$

This is a simple consequence of a Scott-type completion argument. It is also an immediate corollary to the no counterexample theorem.

We adopt for the most part the notation and terminology of [8] and [9].

Types τ have the form $\tau(1) \to (\ldots(\tau(t) \to 0)\ldots)$.
If $\mathcal{S}$ is a set of objects (terms, functionals, etc.), $\mathcal{S}^\tau$ is the set of all members of $\mathcal{S}$ of type τ.

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