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Stationary Logic and Its Friends — II

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Introduction This paper is the successor to "Stationary Logic and Its Friends — I" [10]. The three sections of the paper can be read independently. The first two sections assume some familiarity with stationary logic, denoted $L(aa)$ (see [2]). The third section concerns a closure operation for abstract logic. There a familiarity with [9] would be helpful.

In the first section we define, for regular λ, the λ-interpretation of $L(aa)$, denoted $L(aa^{\lambda})$. In this notation, the standard interpretation is $L(aa^{\omega})$. The most easily understandable case occurs when $\lambda^{<\lambda} = \lambda$. Then for models with universe λ^+, aa^{λ} expresses "for all but a nonstationary set of ordinals of cofinality λ". We show if $\lambda^{<\lambda} = \lambda$, then $L(aa^{\lambda})$ has the same validities as $L(aa^{\omega})$ and $L(aa^{\lambda})$ is (λ, ω)-compact.

The second section is devoted to the proof of the consistency of the following approximation to the Δ-closure of $L(Q)$ being contained in $L(aa)$.

> Suppose $L_1 \cap L_2 = L_0$, $\psi_1 \in L_1(Q)$ and $\psi_2 \in L_2(Q)$.
> Further suppose every finitely determinate L_0-structure *either* can be expanded to a model of exactly one of ψ_1 of ψ_2 *or* can be expanded to a finitely determinate model of exactly one of ψ_1 or ψ_2.
> Then there is a sentence $\theta \in L_0(aa)$ such that every finitely determinate model of ψ_1 satisfies θ and no finitely determinate model of ψ_2 satisfies θ. (So θ separates the reducts of finitely determinate models of ψ_1 from those of ψ_2.)

(See Section 2 for the definition of finitely determinate. Of course Q is the quantifier expressing "there exist uncountably many".) In [10] we showed that every consistent $L(Q)$-sentence has a finitely determinate model. So this result establishes the consistency of the Δ-closure of $L(Q)$ being contained in $L(aa)$

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