Notre Dame Journal of Formal Logic
Volume 33, Number 2, Spring 1992

On Generic Structures

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Abstract We discuss many generalizations of Fraissé's construction of countable 'homogeneous-universal' structures. We give characterizations of when such a structure is saturated and when its theory is ω-categorical. We also state very general conditions under which the structure is atomic.

1 Introduction In this paper we investigate variations on the classical construction of countable homogeneous-universal structures from appropriate classes of finite structures. The most basic result here is the following theorem of Fraissé [1]:

Theorem 1.1 *Let K be a class of finite structures in a finite, relational language that is closed under isomorphism and substructure. Assume further that K satisfies the joint embedding property and amalgamation. Then,*

1. *there is a unique, countable $\mathfrak{A}$ which is "homogeneous-universal" for K, i.e., $\mathfrak{A}$ is (ultra)-homogeneous and K is precisely the class of finite structures embeddable in $\mathfrak{A}$;*
2. *the complete theory of the structure $\mathfrak{A}$ in* (1) *is ω-categorical.*

It is easy to see that (1) holds also for countably infinite relational languages provided K contains only countably many isomorphism types, but (2) may fail in this context. If K is not closed under substructure then the same basic argument establishes a variant of (1) in which $\mathfrak{A}$ satisfies a weaker sort of homogeneity (called *pseudo-homogeneity* by Fraissé); here too (2) may fail, even if the language is finite. More recently, Hrushovski [3,4] has used a construction that generalizes the basic construction by replacing substructure by stronger relations.

In this paper we unify all of these variations in a single framework (allowing also functions and constants in the language). We refer to the resulting structures as *generic* rather than as homogeneous-universal. We then investigate some properties of these generics. Ever since Morley-Vaught there has been a tendency to view homogeneous-universal structures as analogues of saturated models. The

Received December 3, 1990; revised January 20, 1992