154
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Book Review

S. Shelah, *Classification Theory and the Number of Non-Isomorphic Models.* North-Holland, Amsterdam, 1990. 705 pages.

This book contains the proof of the main theorem to date in the branch of model theory known loosely as "Classification theory". The result in question is

The Main Gap Theorem *Let T be a countable first-order theory.*
(1) *If T is not superstable or (is superstable) deep or with the* DOP *or the* OTOP, *then for every uncountable* λ, $I(\lambda, T) = 2^{\lambda}$.
(2) *If T is shallow superstable without the* DOP *and without the* OTOP, *then for every* $\alpha > 0$, $I(\aleph_\alpha, T) < \beth_{\omega_1}(|\alpha|)$.

In this review I will attempt to define some of the terms in the theorem, give a rough outline of the proof (which is several hundred pages long), and explain why Shelah sees this as a completion of the classification problem for countable first-order theories. Although this revised edition was not published until 1990, Shelah has not included any results since 1983 in the main body of the book. I will try to indicate where doing so can simplify the proof. I am assuming in this review that the reader understands the basic notions of model theory (as found in, e.g., Chang and Keisler [3]).

The book contains thirteen chapters, of which nine appeared in the original 1978 edition. As there were no essential changes made to the original chapters I will say little about them. (The material is well-described in Lascar [7] and Baldwin [1].) There are other sources for many of the results contained in the first five chapters reflecting changes in viewpoint which have come about in the past ten years. (See, e.g., Lascar [8], Baldwin [2], Poizat [10], Pillay [9] and Hrushovski [6].) Chapter IV on isolation relations contains many results which will probably not be used elsewhere. There are now a handful of isolation relations which suffice in all known settings. Chapter VI on ultraproducts does not play a role in the proof of the Main Gap Theorem. Chapters VII and VIII contain the so-called many-model arguments. Assuming that the theory somehow codes an order or a complicated tree, the maximum number of models is constructed in any sufficiently large cardinality. The proofs of these theorems are probably the least known of all important results in model theory. The proofs often involve com-

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