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The Gentzen–Kripke Construction of the Intermediate Logic LQ

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Abstract The Gentzen-Kripke construction of the semantics for intermediate logic LQ, which is obtainable from the intuitionistic propositional logic H by adding the weak law of excluded middle $\neg A \vee \neg\neg A$, is presented. Our construction spans the Gentzen system and the Kripke semantics for LQ by providing the way from the cut-elimination theorem to model-theoretic results. The completeness and decidability theorems are shown in this method.

1 Introduction An intermediate logic LQ is obtainable from the intuitionistic propositional logic H by adding the weak law of excluded middle $\neg A \vee \neg\neg A$. The logic can also be axiomatized by the axiom schema $(\neg A \rightarrow \neg B) \vee (\neg B \rightarrow \neg A)$. The motivation of LQ is purely technical rather than philosophical in that LQ is one possible extension of H. Our interest thus lies in its characterization in relation to the formalization of H.

The semantics for LQ can be given by the class of directed Kripke frames; see Gabbay [4]. Several authors also proposed the Gentzen systems for LQ, none of which is successful. For example, the cut-elimination theorem cannot be proved in the sequent calculus of Boričić [2] as Hosoi pointed out. Recently, Hosoi [6] gave a Gentzen-type formulation GQ for LQ and proved the cut-elimination theorem.

The purpose of this paper is to develop the Gentzen–Kripke construction of the semantics for LQ using the subformula models, as a modification of the one developed in Akama [1] for intuitionistic predicate logic. The proposed construction spans the Gentzen system and the Kripke semantics for LQ by providing the way from the cut-elimination theorem to model-theoretic results. The completeness and decidability theorems are shown in this method.

2 Intermediate logic LQ and its Gentzen-type formulation GQ The intermediate logic LQ is one of the extensions of the intuitionistic propositional logic H with *the weak law of excluded middle*, i.e. $\neg A \vee \neg\neg A$. The proof and model

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