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Coherence in Category Theory and the Church–Rosser Property

C. BARRY JAY*

Abstract Szabo's derivation systems on sequent calculi with exchange and product are not Church–Rosser. Thus, his coherence results for categories having a symmetric product (either monoidal or cartesian) are false.

Introduction Gentzen's sequent calculi (Szabo [9]) have been applied extensively in category theory (e.g., Kelly and MacLane [2], Lambek [3] and [4], MacLane [6], Minc [7] and [8]). Sequents correspond to morphisms of a category, and the rules of the calculus correspond to categorical structures (e.g., having an associative tensor product). Cut-elimination was then used to put bounds on the complexity of these structures, e.g. to produce exhaustive lists (perhaps with duplications) of the canonical natural transformations between given functors. For symmetric, monoidal closed categories it was shown in Voreadou [12] how to decide in principle whether two such transformations are equal, whereas an effective, linear-time decision procedure was given in Jay [1].

Derivation systems (reduction rules) can be used to eliminate some duplicates in the list of cut-free proofs (e.g. [8]). However, in *Algebra of Proofs* [11] and its forerunner [10], Szabo claims to have produced derivation systems in which *all* duplicates have been eliminated, so that every proof has a unique normal form. In fact, none of his systems which include a symmetry for the product are Church–Rosser (confluent).

The two major applications of his work were complete coherence theorems for symmetric, monoidal closed categories (mentioned above) and for cartesian closed categories. The latter problem is solved in Lambek and Scott [5] using similar methods, but they avoid adopting symmetry as a primitive by exploiting the universal property of the cartesian product.

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