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Minimal Satisfaction Classes with an Application to Rigid Models of Peano Arithmetic

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Abstract For each regular κ, models of Peano Arithmetic are constructed which are rigid, recursively saturated, and κ-like. The construction relies on a theorem asserting that countable, recursively saturated models of PA have many minimal, inductive satisfaction classes.

After Kaufmann's rather classless model, i.e. an ω_1-like recursively saturated model of PA all of whose classes are definable (cf. Kaufmann [1]), other examples of ω_1-like recursively saturated models of PA, with properties different from those of countable recursively saturated models, are no longer surprising. However, it is still worthwhile to investigate questions about the existence of ω_1-like recursively saturated models with various second-order properties. One reason is that questions about ω_1-like models can usually be translated to questions about their countable elementary initial segments, and these questions often turn out to be interesting in their own right.

In this paper we construct an ω_1-like recursively saturated model of PA which is rigid (that is, it has no nontrivial automorphisms) and even has no nontrivial elementary embeddings into itself. A theorem asserting the existence of rigid ω_1-like recursively saturated models of PA was stated, without proof, in Kossak and Kotlarski [3] as a corollary of a result about automorphisms of countable recursively saturated models. That construction depended on the set-theoretic principle $\diamondsuit$. The construction presented here is based on a MacDowell–Specker type argument, using minimal inductive satisfaction classes, and needs no set-theoretic assumptions. We use it in Theorem 10 and Corollary 11 to construct rigid, κ-like recursively saturated models for all uncountable κ.

A satisfaction class S for a model M is minimal if (M, S) has no proper elementary substructures. We will prove a theorem showing the existence of many minimal inductive satisfaction classes for countable models of PA. A slightly weaker version of this result was stated first without proof in Kossak [2].

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