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Book Review

Stephen Pollard, *Philosophical Introduction to Set Theory*. University of Notre Dame Press, 1990. 180 pages.

What is a set? If Pollard's arguments are accepted, there should be no further need for philosophers of mathematics to pursue this question. This is because he argues that the two viable contenders for the title "mathematically adequate philosophy of set theory" are formalism and structuralism and for neither of these is this a question which it is meaningful to pursue. It would seem that Pollard favors structuralism, but he recognizes that he has not conclusively refuted his own arguments in favor of taking formalism seriously. Pollard's approach to the philosophical problems posed by set theory, whether one agrees with his conclusions or not, deserves careful consideration; he places many issues in a fresh light.

The book begins by asking why the attention of the philosopher of mathematics should be focused on set theory. Pollard's answer appeals to the special foundational role which set theory plays in twentieth century mathematics: "It is the primary mechanism for ideological and theoretical unification in modern mathematics." He does note that set theory is not the only contender for this role. Category theory can also assume a foundational role; it too can be used to supply a unitary and coherent vision of the mathematical enterprise. But, Pollard argues, set theory currently rules the mathematical roost. Even though category theory has shown that it is a viable contender, it has not decisively demonstrated its superiority. Be this as it may, I would suggest that if Pollard's arguments for structuralism succeed, and if they were then followed to their logical conclusion, a re-assessment of category theory would be required. Category theory arose precisely out of a realization that the concepts and constructions which most frequently arise in connection with mathematical structures possess a universality which is independent of their set theoretic origin. Category theory then gives precise expression to the idea that the essence of a mathematical structure is to be sought, not in its internal constitution as a set theoretic entity, but in the form of its relationships with other structures. Its claim to be a superior unifying language for mathematics is based on the fact that it gives direct expression to the centrality of form and structure in mathematics, whereas set theory can express this only indirectly. If we were to become convinced that every mathematical sen-

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