

Relevance and Paraconsistency — A New Approach. Part III: Cut-Free Gentzen-Type Systems

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Abstract The system RMI is a purely relevance logic based on the intuitive ideas of relevance domains and degrees of significance. In this paper, we show that unlike the systems of Anderson and Belnap, RMI has a corresponding cut-free, Gentzen-type version. This version manipulates *hypersequents* (i.e. finite sequences of ordinary sequents), and no translation of those hypersequents into the language of RMI is possible. This shows that RMI is multiple-conclusioned in nature and hints on possible applications of it to the study of parallelism.

I Introduction and background The systems RMI and RMI_{\min} are powerful, purely intentional relevance logics that were introduced in [4]. Semantically they correspond to the algebraic structures which have been developed in [3] following the intuitive ideas of relevance domains, relevance relations, and degrees of reality (or of significance). Our main goal in this paper is to show that, unlike the systems of Anderson and Belnap in [1], RMI and RMI_{\min} have corresponding cut-free, Gentzen-type versions. The existence of such versions is significant from the proof-theoretical point of view and has obvious importance for the task of developing automated reasoning systems that will be sensitive to considerations of relevance and paraconsistency. Even more important, perhaps, is the fact that in the case of RMI, the corresponding Gentzen-type version manipulates *hypersequents* (i.e., finite sequences of ordinary sequents) rather than ordinary sequents. Unlike the case of RM (which we pursued in [5] using similar techniques), in the present case no translation of hypersequents into sentences of the language is possible. Together with the results of section E of [4]

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