

Some Syntactical Properties of Intermediate Predicate Logics

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Abstract In a previous paper the author introduced a syntactical property, which he calls the pseudo-relevance property, for the sake of studying a certain semantical aspect. An intermediate predicate logic \mathbf{L} is said to have the *pseudo-relevance property* if for all formulas A and B which contain no predicate variable in common, either $\neg A$ or B is provable in \mathbf{L} whenever A implies B is provable in \mathbf{L} . The pseudo-relevance property can be regarded as a weak version of Craig's interpolation property. From the same point of view, one can see the similarity between Hallden-completeness and the disjunction property. We treat these syntactical properties and their weak versions, and study the relationships between them.

Introduction In [1], Komori proved that every intermediate *propositional* logic \mathbf{L} has the property that for all formulas A and B which have no propositional variables in common, $A \supset B \in \mathbf{L}$ implies either $\neg A \in \mathbf{L}$ or $B \in \mathbf{L}$. The author [4] showed that the situation changes in intermediate predicate logics. In fact, there are uncountably many intermediate predicate logics without (the predicate version of) the above property. We call here this property the *pseudo-relevance property* (PRP). It is easily seen that PRP can be regarded as a weak version of Craig's interpolation property (see, e.g., Ono [3]). From the same point of view, one can see the similarity between Halldén-completeness and the disjunction property; namely, \mathbf{L} is said to be H-complete if for all formulas A and B which have no predicate variables in common, $A \vee B \in \mathbf{L}$ implies either $A \in \mathbf{L}$ or $B \in \mathbf{L}$. Wroński [7] gave a necessary and sufficient condition for the H-completeness of an intermediate propositional logic by making use of alge-

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