Notre Dame Journal of Formal Logic
Volume 31, Number 4, Fall 1990

Expressiveness and Completeness of an Interval Tense Logic

YDE VENEMA

Abstract We present the syntax and semantics of an interval-based temporal logic which was defined by Halpern and Shoham. It is proved that this logic has a greater capacity to distinguish frames than any temporal logic based on points and we show that neither this nor any other finite set of operators can be functionally complete on the class of dense orders. In the last part of the paper we give sound and complete sets of axioms for several classes of structures. The methods employed in the paper show that it is rewarding to view intervals as points in a plane, in the style of two-dimensional modal logic.

1 Introduction Recent years of research in temporal logic have shown an increasing tendency to concentrate on intervals or events rather than on points. In contrast with the point-based approach, few results are known for modal logics of intervals (see van Benthem [3], Humberstone [11], and Roper [14]), as most authors confine themselves to classical logic. In [10] Halpern and Shoham present a modal logic of intervals which is investigated here. In the first section we define the syntax and semantics of the system and give some basic facts; in Section 2 we present our results on the expressive power of this and other modal logics of intervals, and in the last part of the paper we treat completeness.

1.1 Syntax HS is a tense logic, the formulas of which are built up using the propositional constants $p, q, r, p_0, p_1, \ldots$, the classical connectives $\neg$ and $\wedge$, and the following modal operators: $\langle B \rangle$, $\langle E \rangle$, $\langle A \rangle$, $\langle \underline{B} \rangle$ $\langle \underline{E} \rangle$, and $\langle \underline{A} \rangle$, which have the following intended readings:

$\langle B \rangle \varphi$ φ holds at a strict beginning interval of the current one
$\langle E \rangle \varphi$ φ holds at a strict end interval of the current one
$\langle A \rangle \varphi$ φ holds at an interval met by the current one, i.e., it begins where the current one ends

Received November 28, 1988; revised February 21, 1989