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Varying Modal Theories

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Abstract The notion of modal theory is extended by accepting the idea that axioms and language itself vary over a plurality of possible worlds. Inference rules involving different worlds are introduced and completeness is proved by using a notion of 'ugly diagram', which is a graphical means of detecting when a family of modal theories has no model.

Models of modal theories are indexed by a plurality of possible worlds equipped with a binary accessibility relation. It seems natural to extend the notion of modal theory by accepting that axioms, and even language itself, vary over a similar structure.

Here is an argument which supports our point of view, as opposed to already existing work on modal model theory (e.g. [1]). Consider a language L for a modal theory in the usual sense (L is constant). Consider a modal structure M: it varies with the elements of a set I. We may define the "theory of M" as the set of sentences satisfied in the "actual world", but we could as well consider for each $i \in I$ the set T_i of sentences satisfied by M_i in L. A further step consists in the adjunction for each $i \in I$ of constants $\underline{a}_i$ for $a_i \in M(i)$, giving rise to languages $L_i = L \cup \{\underline{a}_i | a_i \in M(i)\}$ varying over the set I of indices.

The aim of this paper is to answer the following preliminary question: when is a family of usual modal theories the theory (in our sense) of a model?

To be specific, we will deal with the system K in the main body of the text but discuss in the last section the extension to other systems.

In the first section, we propose a notion of (K-) theory $(T_i)_{i \in I}$ varying over a structure $\langle I, R \rangle$. Structures and models for these theories are essentially the usual ones (see e.g. [3]), but we note that models validate rules of deduction involving different indices. To take a simple example: if a sentence $\Box\varphi$ is satisfied in i and if iRj, then φ is satisfied in j.

In the second section we describe a notion of consistency. It is clearly necessary but not sufficient to say that for each $i \in I$, T_i is K-consistent; if T_i proves $\Box\varphi$ in K, if iRj and T_j proves $\neg\varphi$ in K, then $T = (T_i)_{i \in I}$ has no model. It is

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