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A Lemma in the Logic of Action

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Abstract In this paper, a result is proved that has two consequences for Segerberg's Logic of Action. First, in [1] and [2] his general frames can be replaced by full frames without change to the logic; secondly, a certain rule in [2] is proved to be sound.

Introduction The ultimate goal of this paper is to show that within the imperative logic described in Segerberg [2] the rule

$$\frac{\vdash [\alpha]\mathbf{P} \equiv [\beta]\mathbf{P}}{\vdash !\alpha \equiv !\beta} \quad \text{where } \mathbf{P} \text{ is a propositional variable not in either } \alpha \text{ or } \beta \qquad \text{[II]}$$

is sound. In showing this we establish a result in the underlying logic of action, namely that Segerberg's restriction of the set of propositions in a frame is unnecessary. Essentially what we will show is that, given a standard frame $\mathfrak{F} = (U, A, D, P)$ with $D : P \to A$ satisfying

(D1) $D(X)(x) \subseteq X$, for all $X \in P$, $x \in U$
(D2) $D(X)(x) \subseteq Y \Rightarrow D(X)(x) \subseteq D(Y)(x)$, for all $X, Y \in P$, $x \in U$

we can find an extension D' of D to the whole of $\mathcal{P}(U)$, where D' still maintains these conditions. With this result, we will be able to show that given any countermodel to $!\alpha \equiv !\beta$ we can construct another model in which $[\alpha]\mathbf{P} \equiv [\beta]\mathbf{P}$ fails to hold.

1 Frames We take as our standard frames those outlined in Segerberg [1]. For a function f with range $\mathcal{P}(U \times U)$ we take $f(X)(x) = \{y : \langle x, y\rangle \in f(X)\}$, and use $f|_P$ to mean the restriction of f to P.

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