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Book Review

John Bell. *Toposes and Local Set Theories, An Introduction.* Oxford Logic Guides 14, Oxford University Press, 1988. 267 pages.

Toposes come to logic from outside, notably from category theoretic methods in geometry, and the border crossing has occasioned its share of difficulties. Most of the present work aims to initiate the newcomer to toposes, connecting them to established concerns in logic. A classical logician and set theorist whose own initial response to toposes in the foundations of mathematics was skeptical (see [2]), Bell introduces topos theory in a way congenial to mainstream logicians and with his usual expository skill. Then he swings away from the project of initiation into a speculative epilogue. He goes beyond the conventional model theoretic content of the body of the book to suggest that abstraction in mathematics, including set theory, has led to a point where the "pluralism" of category theory must replace the "monism" of set theory (p. 235). His arguments here should provoke debate from all sides.

Requiring basic knowledge of model theory and set theory, the book covers the standard theorems of topos theory and some set theoretic techniques for constructing toposes. It requires no prior knowledge of category theory, although a reader might find it helpful to look at some of the other sources Bell cites or at the brief nontechnical treatment in [14]. The book gives more extensive and elementary treatment of logic in toposes than [9], and could serve as an introduction to that book's results on categorical methods in the lambda calculus and recursive functions. It could also serve a logician as a starting point toward understanding Lawvere's work, and also towards [1], [6] and other research literature in toposes and categorical logic, although for these latter it would have to be supplemented by more general category theory.

A topos can be seen as a kind of universe in which one can interpret higher-order logic and do mathematics. The universe of sets is an example and so are its Boolean extensions as used in independence proofs. In a less classical vein there is a well-known *topos of smooth spaces*: A universe which includes among its objects a line R, a plane R^2, and so on through all classical manifolds of differential geometry and more, including infinitesimal spaces. In this topos every object has a geometric structure and every function is continuously differentiable. One can work within this topos more or less as if working with ordinary sets and arbitrary functions, and yet be assured that all the functions one con-