

Modal Definability in Enriched Languages

VALENTIN GORANKO*

Abstract The paper deals with polymodal languages combined with standard semantics defined by means of some conditions on the frames. So a notion of “polymodal base” arises which provides various enrichments of the classical modal language. One of these enrichments, viz. the base $\mathcal{L}(R, -R)$, with modalities over a relation and over its complement, is the paper’s main paradigm. The modal definability (in the spirit of van Benthem’s correspondence theory) of arbitrary and Δ -elementary classes of frames in this base and in some of its extensions, e.g., $\mathcal{L}(R, -R, R^{-1}, -R^{-1})$, $\mathcal{L}(R, -R, \neq)$ etc., is described, and numerous examples of conditions definable there, as well as undefinable ones, are adduced.

Introduction Undoubtedly, first-order languages are reliable and universal tools for formalization. However, in some cases the cost of this universality is not fully acceptable: on the one hand we have the undecidability results, and on the other the fact that the expressive power of first-order languages does not allow any possibility for a categorical characterization of a given infinite model since it is elementarily equivalent to any of its ultrapower. So it is desirable, sometimes even necessary, to seek alternative languages for particular types of

*Research partially supported by the Committee for Science at the Council of Ministries of Bulgaria, Contract No. 56.

I am grateful to Dimiter Vakarelov, my first teacher in modal logic. The present investigation was inspired and actively countenanced by Solomon Passy, and I express my great thanks to him. Special thanks are due to George Gargov, who made some valuable remarks on the paper and whose talks helped me to orient myself in certain things. I also thank the anonymous referee of the first version of the paper for the useful criticism and the note, simplifying the proof of 3.5, and Mark Brown from Syracuse University for some stylistic suggestions.