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# Linear Logic Displayed

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**Abstract** "Linear logic" (**LL**; see Girard [6]) was proposed to be of use in computer science, but it can be formulated as a "display logic" (**DL**; see Belnap [2]), which is a kind of Gentzen calculus admitting easy proof of an Elimination Theorem. Thus **LL** is naturally placed within a wider proof-theoretical framework that is known to include relevance, intuitionist, and modal logics, etc., and that permits coherent variations on **LL** itself—including the definition of "punctual logic". In order to accommodate **LL**, two independently useful modifications of **DL** are made. First, **DL** possessed an unmotivated identification of two of its structural constants. This identification is dropped in order to make room in **DL** for the several propositional constants in **LL**. Second, **DL** possessed an unmotivated bias towards connectives that, when they are introduced as consequents, have restrictions put on their antecedents. This bias is abandoned in order to make room in **DL** for a dual pair of modal-like "exponential" connectives of **LL**. The latter modification requires restructuring the proof of the Elimination Theorem for **DL**, rendering it perfectly symmetrical in antecedent and consequent.

*1 Introduction* To "display" any logic is to exhibit it as a display logic, that is, as the special sort of Gentzen consecution calculus defined in Belnap [2] (**DL**), and thus to place that logic within a certain significant proof-theoretical framework. The aim of this paper is to display "linear logic", which is a logic proposed by Girard [6] in connection with some important computer science considerations.[1] It turns out that the display of linear logic requires some healthy adjustments in the universal features of display logic itself.[2] One set of adjustments is required in order to treat the "exponentiation" connectives of linear logic, and another to treat its four (instead of two) propositional constants while keeping to a single "family" of display logic. After we are done, we will be able to see how displaying linear logic permits a well-organized consideration of some of its essential features and of some of its variants.

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