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On Finite Models of Regular Identities

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Abstract It is a known result of Austin that there exist nonregular identities with all nontrivial models being infinite. In this note a certain analogue of this result for regular identities is presented and some remarks in this connection are given.

1 Perkins [4] proved that it is undecidable whether an identity (and in consequence, a finite set of identities) has a nontrivial model (i.e., a model of cardinality greater then one). Austin [1] in improving the result of Stein [5] found an identity with infinite models but with no nontrivial finite one. McKenzie [3] proved that it is also undecidable whether an identity (a finite set of identities) has a nontrivial finite model.

All these results are based on properties of nonregular identities. For regular identities (i.e., those with the same variables appearing on both sides, cf. [2]) the model problems mentioned above are trivial. It is known that for each finite set of regular identities the so-called τ-semilattices provide models of arbitrary cardinalities.

More precisely, let τ be a finite type of algebras, i.e., a sequence $\langle n_1, \ldots, n_k \rangle$ of nonnegative integers, and xy a semilattice operation on a set A (a semilattice operation can be defined on every finite set A). For $1 \leqq i \leqq k$ we define an n_i-ary operation on the set A by $f_i = x_1 x_2 \ldots x_{n_i}$. The algebra $\langle A, f_1, .., f_k \rangle$ is then called a *τ-semilattice*. Any τ-semilattice is polynomially equivalent to the corresponding semilattice and clearly is a model for any set of regular identities in type τ. Let us also note that each one-element algebra is a τ-semilattice.

Thus, τ-semilattices can be treated as trivial models for regular identities. Let us now inquire about other models.

We will show that a set of regular identities close to the lattice identities

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