

Paraconsistency and the C-Systems of da Costa

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Abstract It is argued that, both in view of da Costa's conditions for paraconsistent systems and for independent reasons, the failure of the C-systems to enjoy the property of intersubstitutivity of provable equivalents constitutes a deficiency which it is reasonable to attempt to remedy. Two extensions of the C-systems are considered, and both are shown to collapse all but the base system C_ω into classical logic. The general result is established that there is no extension of the stronger C-systems which both enjoys the intersubstitutivity property and is weaker than classical logic. Methods of constructing alternative but related hierarchies for which this property might more successfully be secured are suggested.

1 The C-systems and the paraconsistency conditions The systems C_n ($1 \leq n \leq \omega$) are among the best-known contributions of da Costa and his collaborators to the program of constructing paraconsistent logics, i.e., logics capable of supporting inconsistent theories without collapse into triviality.

In [5], da Costa and Alves state that, in general, systems of paraconsistent logic must satisfy the following conditions:

- (I) From two contradictory formulas A and $\neg A$, it must not be possible in general to deduce an arbitrary formula B
- (II) Such systems should contain most of the schemata and deduction rules of classical logic that do not interfere with (I).

In considering the C-systems specifically, da Costa states in [4] that "it seems natural that they satisfy" not only (I) and (II), but also the following:

- (III) In these systems, the schema $\neg(A \ \& \ \neg A)$ must not be derivable
- (IV) It must be simple to extend the systems to first-order predicate calculi (with or without equality).

Of these four conditions, only two are beyond controversy: (I) is unanimously accepted as a necessary condition for paraconsistent systems, and (IV) is also uncontroversial, if only because paraconsistency researchers have stan-