Notre Dame Journal of Formal Logic
Volume 30, Number 4, Fall 1989

Near Coherence of Filters III: A Simplified Consistency Proof

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Abstract In the model obtained from a model of the continuum hypothesis by iterating rational perfect set forcing $\aleph_2$ times with countable supports, every two nonprincipal ultrafilters on ω have a common image under a finite-to-one function.

The principle of near coherence of filters (NCF) asserts that, for any two nonprincipal ultrafilters $\mathcal{U}$ and $\mathcal{V}$ on the set ω of natural numbers, there exists a finite-to-one function $f: \omega \to \omega$ such that $f(\mathcal{U}) = f(\mathcal{V})$. This principle was introduced and studied in [1], and its consistency relative to ZFC was proved in [2]. Because [2] also contains the consistency proof for another statement (the existence of simple P_κ-points for two different κ), the model of NCF presented there was chosen to maximize the similarity of the two proofs. Although this approach is quite efficient for proving the consistency of both statements, there is a simpler consistency proof for NCF alone. The purpose of this paper is to present this proof.

By *rational perfect set forcing*, we mean the forcing introduced by Miller in [3]; a definition is given below.

Theorem NCF *holds in the model obtained from a model of the continuum hypothesis by iterating rational perfect set forcing* $\aleph_2$ *times with countable supports.*

The proof to be presented here can be viewed as the result of deleting, from the proof in [2], all references to (what is there called) depth. The observation that the consistency proof for NCF survives this deletion was made by Shelah shortly after he found the proof in [2]. Blass noticed that the resulting forcing was equivalent to Miller's rational perfect set forcing.

The substitution of rational perfect set forcing for the forcing used in [2]

Received May 5, 1986