1 Introduction

A K-modal logic based on Łukasiewicz’s three-valued logic has been formulated by Schotch [2]. In this paper we formulate K-, M-, S4-, and S5-modal logics based on a general three-valued logic by using the notion of a matrix in [3].

In Section 2, we define truth values, formulas, and matrices. In Section 3, we introduce three-valued Kripke models defined in [1]. In Section 4, we present the systems K, M, S4, and S5 of modal logic based on a general three-valued logic (3-K3, 3-K2, 3-M3, 3-M2, 3-S43, 3-S42, 3-S53, and 3-S52). 3-K3, 3-M3, 3-S43, and 3-S53 are modal logics based on a three-valued logic in which the modal operators take on all three of our truth-values. 3-K2, 3-M2, 3-S42, and 3-S52 are modal logics based on a three-valued logic in which the modal operators take only the two classical truth-values. In Section 5, we develop the syntax of 3-K3, 3-M3, 3-S43, and 3-S53 (i = 2, 3) and it will be shown that the cut-elimination theorems no longer hold in 3-K3, 3-M3, 3-S43, and 3-S53. In Section 6, we prove the completeness theorems for 3-K3, 3-M3, 3-S43, and 3-S53.

2 Matrices

2.1 Truth values

We take 1, 2, and 3 as truth-values. Intuitively ‘1’ stands for ‘true’ and ‘3’ for ‘false’, whereas ‘2’ may be interpreted as ‘undefined’ or ‘meaningless’.

We denote the set of all the truth values by T. T = {1, 2, 3}.

2.2 Primitive symbols

(1) Propositional variables: p, q, r, etc.
(2) Propositional connectives:

\[ F_i(\ast_1, \ldots, \ast_{\alpha_i}) = i = 1, 2, \ldots, \beta, \alpha_i \geq 1. \]