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Book Review

Stewart Shapiro. *Foundations Without Foundationalism, A Case for Second-Order Logic*. Oxford University Press, Oxford, 1991.

This is an excellent book, covering of all of the main results in second-order logic and its applications to mathematical theories. Its main theme is that first-order logic does not adequately "codify the descriptive and deductive components of actual mathematical practice," and that "first-order languages and semantics are also inadequate models of mathematics" (43). Second-order logic (under its "standard" semantics), Shapiro maintains, "provides better models of important aspects of mathematics, both now and in recent history, than first-order logic does" (v); and in that regard it is second-order, and not only first-order, logic that "has an important role to play in foundational studies" (ibid.). Indeed, the restriction of logic to first-order logic (without Skolem relativism) in such studies is "the main target of this book" (196).

The book is divided into three parts, with Part I containing a discussion of the philosophy of logic and the role of logic in foundational studies. In Part II, a technical development of second-order logic is given in which it is argued that "higher-order notions are well-suited for modelling important aspects of mathematics" (x). Additional philosophical issues relevant to the acceptance of second-order logic are given in Part III, as well as a coverage of some of the history of these issues in the early part of this century.

1 Full logics By a *full logic*, Shapiro means "a [formal] language, together with a deductive system and a semantics" (3), where the language is taken as a model of a fragment of ordinary natural language considered as a natural language of mathematics. Given a set K of nonlogical symbols (individual, function, and predicate constants), the main full logics considered here are $L1K{=}$, first-order logic with identity, and $L1K$, first-order logic without identity, respectively, as based on the symbols in K (with $\rightarrow$ for the material conditional, $\neg$ for classical negation, $\forall$ for universal quantification, and $\exists$ and other truth-functional connectives defined in the usual way). $L2K-$ is $L1K{=}$ extended to contain predicate and function variables of different degrees, but no quantifiers binding these; and $L2K$ is $L1K$ similarly extended, but with quantifiers binding predicate and function variables as well as individual variables.

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