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Some Admissible Rules in Nonnormal Modal Systems

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Abstract Epistemic logics for subjects of bounded rationality are in effect nonnormal modal logics. Admissible rules are of interest in such logics. However, the usual methods for establishing admissibility employ Kripke models and are therefore inappropriate for nonnormal logics. This paper extends syntactic methods for a variety of rules (e.g. the rule of disjunction) and nonnormal logics. In doing so it answers a question asked by Chellas and Segerberg.

1 Introduction The admissibility of a rule by a logic depends only on the logic's set of theorems. It does not depend on a choice of semantics or proof system (for which reason the phrase "rule of proof" is not ideal; but see Humberstone [4]). However, the usual methods of proving the admissibility of a rule in modal logic are semantic; they use standard "possible worlds" model theory. This semantic treatment is applicable only to normal model systems (see below). Thus the usual methods do not allow one to prove the admissibility of a rule in a nonnormal modal system. The aim of this paper is to extend the use of syntactic methods for proving admissibility, methods applicable to both normal and nonnormal systems.

An important example is the *rule of disjunction*. A system S provides (admits) this rule just in case for all formulas $A_1, \ldots, A_n$:

$$\text{if } \vdash_S \Box A_1 \vee \ldots \vee \Box A_n$$
$$\text{then } \vdash_S A_i \quad \text{for some } i \quad (1 \leq i \leq n).$$

Lemmon and Scott established the rule of disjunction for a variety of modal systems by a model-theoretic technique that is now standard (Lemmon and Scott [9] pp. 44–46 and 79–81; Chellas [1] pp. 181–182; Hughes and Cresswell [3] pp. 96–100; see also Kripke [5], Lemmon [8], McKinsey and Tarski [10] and Segerberg [13]). Powerful though such techniques are, they are restricted to systems amenable to the model theory in question. Thus if a modal logic is nonnormal, because it lacks the rule

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