Notre Dame Journal of Formal Logic
Volume 34, Number 3, Summer 1993

The Fibrational Formulation of Intuitionistic Predicate Logic I: Completeness According to Gödel, Kripke, and Läuchli, Part 1

M. MAKKAI

Abstract Following the pattern of Lawvere's notion of hyperdoctrine, we single out certain classes of fibrations and use them, in the present paper and its sequel, to give an algebraic framework for the proof theory of intuitionistic predicate calculus. The two papers are organized around representation theorems that correspond to and strengthen the completeness theorems of the title. The present first part deals with the fibrational analog of Gödel's completeness theorem and gives fibrational liftings of well-known categorical constructions. The present first part is a preparation for the main results to be given in the second part.

0 Introduction This is an introduction to both the present first part and the second part of the paper (cf. Makkai [18]). The numbering of the sections of the second part continues that of the first part; Sections 1 to 3 form the first part, Sections 4 to 6 the second part.

By the "fibrational formulation of predicate logic" I mean the approach to predicate logic using the notion of hyperdoctrine, originally introduced by Lawvere in [15],[16], and its variants. Although Lawvere, and other authors following him (see below), used pseudofunctors as the basic ingredients, in this paper I adopt the essentially equivalent language of fibrations; hence the reference to fibrations in the title.

A fibration is given by two categories $\boldsymbol{B}$ and $\boldsymbol{C}$, the base category and the total category, respectively, of the fibration, together with a functor $\boldsymbol{C} \to \boldsymbol{B}$; certain conditions have to be satisfied by these data for them to form a fibration; at this point, it is not important to know what these conditions are. In the fibrational formulation of logic, a(n axiomatic) theory is construed as a fibration. This is in contrast to the more widely discussed categorical approach to logic in which a theory is made into a (single) category (with appropriate properties). From the point of view of the present paper, the reason for adopting the more elaborate approach of fibrations is that it has the capability of incorporating the notion of proof (formal deduction), and not just

Received August 8, 1989; revised February 28, 1991