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Remarks on Strong Nonstructure Theorems

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Abstract In this paper we continue the work started in Hyttinen and Tuuri [4]. We study the existence of universal equivalence trees and the existence of strongly bistationary sets.

1 Introduction In this paper we answer some questions left open in [4]. The starting point in [4] was Shelah's nonstructure theorem for unstable (and DOP and OTOP) theories. In [4] we looked how strong nonstructure theorems can be proved in terms of Ehrenfeucht-Fraisse games (see below). In many cases we were able to prove maximal results by using rather strong cardinal assumptions, but also many questions were left unanswered. In this paper we answer two of those questions.

One case in which we studied strong nonstructure theorems in [4] was the case in which we assumed about the theory only that it is unsuperstable. In this case we cannot prove maximal results, as shown in [4]. The theorems we were able to prove depend on the existence of so-called strongly bistationary sets. In the first part of this paper we continue the studies on the existence of these sets. In the main result of this part we show that if $\lambda = \kappa^+$, $\kappa > \xi \geq \omega$, $cf(\kappa) < \kappa$ and $A \subseteq \{\alpha < \lambda \mid cf(\alpha) = \xi\}$ is stationary then there is strongly bistationary $B \subseteq A$.

In the second part of this paper we study the existence of universal equivalence trees (see Definition 2.3).

In Hyttinen and Shelah [2] and [3] we will continue the studies of strong nonstructure theorems in the case the theory is unsuperstable.

2 Preliminaries In this chapter we give the most important definitions and a theorem from [4] needed in this paper. The proof of the theorem in [4] is based on the construction of Shelah in [9].

Definition 2.1 Let λ be a cardinal and α an ordinal. Let t be a tree (i.e., for all $x \in t$, the set $\{y \in t \mid y < x\}$ is well-ordered by the ordering of t). If $x, y \in t$ and $\{z \in t \mid z < x\} = \{z \in t \mid z < y\}$, then we denote $x \sim y$, and the equivalence class of x for $\sim$ we denote $[x]$. By a λ,α-tree t we mean a tree which satisfies:

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