542
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On the Proofs of Arithmetical Completeness for Interpretability Logic

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Abstract Visser proved that **ILP** is the interpretability logic of any finitely axiomatizable theory containing $I\Delta_0 +$ SUPEXP, Berarducci and Shravrukov proved that **ILM** is the interpretability logic of PA. But these proofs are not based directly on the natural semantics of interpretability logic (i.e., Veltman models). We give simpler alternative proofs of the arithmetical completeness of **ILP** and **ILM** directly based on finite Veltman models. We will provide a general setup for arithmetical completeness proofs of interpretability logic which is in the style of Solovay's arithmetical completeness proof of provability logic.

1 Introduction Visser [7] introduced the binary modal logic **IL** (interpretability logic) and its extensions **ILM** (interpretability logic with Montagna's axiom) and **ILP** (interpretability logic with a persistent relation in its models) to describe the interpretability logic of PA and the interpretability logic of any sufficiently strong theory T which is finitely axiomatizable and Σ_1 sound. The modal completeness of **IL**, **ILP**, and **ILM** was provided by de Jongh and Veltman [3] using so-called Veltman models. These are a very natural generalization of Kripke models. Visser [8] obtained the arithmetical completeness for **ILP** and, more recently, Berarducci [1] and Shavrukov [5] have shown **ILM** to be complete for arithmetical interpretation over PA. All these proofs of arithmetical completeness do not directly use the Veltman models. Using a bisimulation, Visser [8] showed **ILP** to be modal complete with respect to his so-called Friedman models and then used these to prove arithmetical completeness. Berarducci and Shravrukov also used a bisimulation due to Visser [7] showing that **ILM** is modal complete with respect to the so-called simplified models to prove arithmetical completeness. The use of simplified models in proving arithmetical completeness for **ILM** adds a complication because in general these cannot be taken to be finite.

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