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Book Review

Richard Kaye. *Models of Peano Arithmetic*. Oxford Logic Guides, Oxford University Press, Oxford. 1991.

Our vocabulary lacks a term to denote a person whose calling is the study of models of arithmetic. Model theorists, topologists, even functional analysts can identify themselves succinctly, but we have to resort to such locutions as "I'm in models of arithmetic". And the name of the field itself—"models of arithmetic"—also seems to bespeak an insecurity about whether it is a field at all: the objects of study are baldly named without any pretensions to a grand theory or -ology.

Models of arithmetic certainly is a bona fide field. It has its own meetings, folklore, and stars. It has built up a coherent body of knowledge relevant to some of the central problems of modern logic. But it has never sat comfortably within the traditional fourfold division of logic, it is sparsely populated and has been known to lie dormant for decades, and it has never had a "bible". Access to this difficult terrain has been daunting to outsiders.

Under these circumstances the appearance of this volume by a leading young modelofarithmetist is a significant event, perhaps marking an overdue coming-of-age of the field. With no previous books on the subject (except conference proceedings), Kaye has bravely stepped into the void, setting himself the difficult task of writing a comprehensible introduction to the subject while including a good amount of material of more advanced interest. He has achieved this admirably well. Although space limitations preclude an encyclopedic work, Kaye presents all the mainstream material straightforwardly and interestingly, and he goes on to reach far into the depths of the subject in certain directions. He finishes with an excellent survey of further reading. Throughout he provides stimulating and challenging exercises.

Kaye's no-frills approach is reflected in the overall plan of the book, which is clear and uncluttered. I shall sketch what he does and offer minor comments.

After presenting the background material that the reader should know—this book will be accessible to a first-year graduate student—Kaye begins with a brief look at the standard model of arithmetic: this is a nice touch. He then introduces the base theory PA^- (essentially the theory of discretely ordered rings), and gives a traditional enough proof of the first incompleteness theorem for PA^- (with the second sketched as an exercise). There are a few reasons why I would