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Unifying Some Modifications of the Henkin Construction

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Abstract This paper is a continuation of the work of Leblanc, Roeper, Thau, and Weaver, which modified the Henkin construction to yield various necessary and sufficient conditions for extending a consistent set of sentences in a countable first order language to a maximally consistent and ω-complete set in that language. In this paper the theory of abstract deducibility relations introduced by Goldblatt is extended to provide an abstract setting for these and related results. Modifications of Henkin construction are replaced by Goldblatt's Countable Henkin Principle to yield abstract forms of the ω-completeness theorem, the soundness and completeness of ω-logic, the theorem to the effect that ω-logic is a conservative extension of standard logic for ω-complete sets, and the theorem that all ω-complete sets are ω-consistent. These abstract results specialize to yield the corresponding "concrete" ones.

The main technical innovation (the characterization of the smallest deducibility relation that respects all members of a family of premise–conclusion arguments and extends a given deducibility relation) is motivated by the observation that the deducibility relation determined by ω-logic is the smallest deducibility relation which extends the deducibility relation determined by standard logic and under which every set of sentences respects the ω-rule.

1 Introduction In Henkin [3] the method of constants was introduced to show that every consistent set of first order sentences has a model. The construction of this model is often called the Henkin construction. Part of the construction involves showing that the consistent set can be extended to a maximally consistent set which contains a universal quantification if it contains all of its instances. This extension is accomplished by adding "enough" new individual constants to the nonlogical vocabulary of the language. This addition cannot, in general, be avoided. It is easily shown that, even when the nonlogical vocabulary of the language contains individual constants, there are consistent sets of

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