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Constructive Ultraproducts and Isomorphisms of Recursively Saturated Ultrapowers

G. C. NELSON

Abstract Various models of a first order theory T are obtained from given models of that theory by generalizations of the ultraproduct construction. It is demonstrated that for a model complete theory this construction can be carried out using as functions for the ultraproduct exactly those functions defined by terms in an extension of the original language. In this way one obtains countable nonstandard models of T which can be endowed with other desirable properties such as being recursively saturated. These constructions use only the most basic ideas of model theory and recursion theory. Two countable elementarily equivalent models are shown to have recursive ultrapowers which are isomorphic and recursively saturated.

0 Introduction and recursive ultraproducts The ultraproduct construction is one of the basic constructions of model theory and its applications are numerous (see Chang & Keisler [1]). It is particularly useful in that it enables one to prove that the notion of two structures in a first order language being elementary equivalent is an algebraic property via the Keisler–Shelah Theorem, which equates this property with that of having isomorphic ultrapowers. Another use of this construction is to show that a given mathematical property of structures is not a first order property by showing that there is an ultraproduct of structures with the given property which does not inherit that property.

Another application of ultraproducts and ultrapowers is the following: in order to attempt to understand what all models of a given first order theory T look like, we might start with some well understood subset of models of T and consider what models can be constructed from them using ultraproducts and ultrapowers. If we had started with an initial class which contained a model of each complete extension of T and had infinite wisdom, then in view of the Keisler–Shelah Theorem we would see all other models of T as an elementary substructure of these models. If we did not have a representation of each complete extension of T initially, then under suitable assumptions we still might be

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