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A Note on Naive Set Theory in LP

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Abstract Recently there has been much interest in naive set theory in nonstandard logics. This note continues this trend by considering a set theory with a general comprehension schema based on the paraconsistent logic **LP**. We demonstrate the nontriviality of the set theory so formulated, deduce some elementary properties of this system of sets, and also delineate some of the problems of this approach.

It has long been a desire among certain logicians that there be a generally satisfactory formalization of the naive theory of sets. Much work has gone into finding such a formalization, and this paper is another attempt to go some way in that direction. The paper is structured in three sections. First, I introduce the logic and the formalization of naive set theory that we will consider. Second, I give formal results concerning this theory—its nontriviality, its relationship to **ZF**, and the existence of empty and universal sets. Finally, I critically evaluate the theory and consider where a naive set theorist might go from here.

1 LP and naive set theory Any study of a theory must involve a choice concerning the logic in which the theory is embedded. The logic of choice for this exercise is the paraconsistent logic **LP**, which we introduce below.

We can define **LP** in various ways. It has the same semantics for connectives and quantifiers as Kleene's 3-valued logic, except that the middle value is designated. Also, it is the '$\rightarrow$' free fragment of the quasi-relevant logic **RM3**. It is also a simple revision of the classical predicate logic, in which a formula is allowed to be evaluated as both true and false. We will officially define it in this way.

Let $\mathcal{L}$ be a first order language with $\wedge$, $\neg$, and $\forall$ the primitive connectives and quantifier, $\in$ a dyadic predicate symbol, and $x, y, z, x_1, x_2 \dots$ the variables. Then a pair $A = \langle D, I\rangle$ is said to be an **LP**-*model structure* if D is some nonempty domain of objects, and for each pair a, b of elements in D, we have $I(a \in b) \in \{\{1\}, \{0\}, \{0,1\}\}$. (We will define T, F, and B to be $\{1\}$, $\{0\}$ and $\{0,1\}$ respectively.)

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