On a Quasi-Set Theory

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Abstract The main features of a theory that enables us to deal, in terms of a set theory, with collections of indistinguishable objects are presented. The fundamental idea is to restrict the concept of "identity" in the underlying logical apparatus. The basic entities of the theory are *Urelemente* of two sorts; to those called *m*-atoms, the usual concept of identity, in a precise sense, does not apply, but there exists a primitive equivalence relation called "the indistinguishability relation" that holds among them. The other sort of atoms (*M*-atoms) are treated as *Urelemente stricto sensu*. The underlying logic is a kind of "nonreflexive logic" and reflects formally this situation. The intuitive motivation is twofold: seeking agreement with Schrödinger's dictum that "identity" lacks sense with respect to the elementary particles of modern physics, and building Weyl's "effective aggregates" "directly", that is, dealing *ab initio* with indistinguishable objects; hence, their collection must not be considered a "set". Despite these motivations, in this paper quasi-set theory is delineated as a set theory, independently of its possible applications to other domains.

1 The intuitive idea of a quasi-set   To understand intuitively what we mean by a *quasi-set* (*qset* for short), the reader may think of a *classical* set with atoms (in the sense of Zermelo–Fraenkel with *Urelemente* – ZFU). Suppose now that the atoms are of two sorts. In the first category we have the *M-atoms*, which can be thought of as the *macroscopic* objects of our environment. They will be treated as *Urelemente* of ZFU *stricto sensu*; hence, we will admit that classical logic is valid with respect to them in all its aspects. The atoms of the other kind (*m-atoms*) may be intuitively thought of as elementary particles of modern physics, and we will suppose, following Schrödinger's ideas, that *identity* is meaningless with respect to them ([10], pp. 16–18).[1] Then we will admit that the Traditional Theory of Identity (TTI) does not apply to the *m*-atoms. These facts enable us to hold, with regard to the *m*-atoms, that the concepts of *indistinguishability* and *identity* may not be equivalent.[2] Therefore, roughly speaking we can say that a *qset* is a collection of objects (called *elements*) such that to the elements