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Cut-Free Modal Sequents for Normal Modal Logics

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Abstract We develop cut-free calculi of sequents for normal modal logics by using semantic modal sequents that are trees of usual sequents plus an accessibility relation, and by introducing modal operators when moving formulas along the branches of such trees. Those calculi are a natural improvement of modal tree-sequents, introduced in a previous work, are cut-free, and work well for all of the main normal modal logics.

0 Introduction We introduce a variant of sequents to develop cut-free calculi for normal modal logics (NLs). Namely, we enhance the modal tree-sequents introduced in Cerrato [3] (as the counterpart of systems of natural deduction based on strict implication developed in Cerrato [2]) by considering trees of usual sequents instead of trees of sequences of formulas, and by adding the Kripkean accessibility relation to those trees (so we call our modal sequents "semantic").

We use only two general modal rules for all NLs $\Box\vdash$ and $\vdash\Box$ (with a technical exception for systems containing the axiom schema D): we vary the first rule when varying the system depending on the accessibility, while we fix the second rule for all NLs.

We prove the completeness of our calculi for the normal modal logics **K**, **KB**, **KD**, **KT** (=**T**), **K4**, **K5**, **KBD**, **KBT** (=**B**), **KB4**, **KD4**, **KD5**, **KD45**, **K45**, **KT4** (=**S4**), **KT5** (=**S5**) (see Chellas [4]), giving also a semantic proof of cut-elimination, and, as a corollary, of the subformula property.

Thus we obtain a uniform treatment of calculi of sequents the normal modal logics that work well in every case. Namely, our calculi work better than the usual ones (Fitting [6], Ohnishi [8], [9]) that are not cut-free for some systems (e.g., for "symmetric" systems, those containing the axiom schema B) even when the subformula property holds (Takano [12]). Furthermore, our calculi work also better than those using higher level sequents (Došen [5]), that are not cut-free, and are developed only for **S4** and **S5**. Finally, our calculi work better than those

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