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Multimorphisms Over Enumerated Sets

ANDRZEJ ORLICKI

Abstract The concept of a multimorphism over enumerated sets is a natural generalization of the classical concept of a morphism of enumerated sets. Moreover, there are some connections between multimorphisms and binary relations over enumerated sets (Orlicki [3]). These connections are presented in the first part of the paper. In the second part the main results of the paper are given. The third part shows us that multimorphisms are not "exotic"—they appear in Recursion Theory.

0 Introduction and preliminaries The fundamental idea of the Theory of Enumerations is the following: computability over arbitrary countable sets is realized as a usual computability over natural numbers by using suitable enumerations of the sets considered. An interesting philosophical interpretation of this idea has been given by Eršov in the introduction to his famous monograph [2]—in his opinion, Theory of Enumeration is a kind of modern version of pythagoreanism. Consequently, we get the well-known concept of a morphism of enumerated sets: Let $\underline{S}_i = \langle S_i, \nu_i \rangle$ $(i = 1,2)$ be two (non-empty) enumerated sets and let $\mu : S_1 \to S_2$ be an arbitrary function. We say that μ is a morphism from $\underline{S}_1$ to $\underline{S}_2$ iff there is a general recursive function f such that $\mu\nu_1 = \nu_2 f$, i.e., there exists an algorithm that "realizes" μ on the level of codes of elements of sets S_1 and S_2. It is clear that f cannot be an arbitrary general recursive function—it must preserve kernels of suitable enumerations. But we do not assume that these kernels are decidable (it is worth noting that in that case Eršov also gave an interesting philosophical motivation). A mathematical practice gives us many important examples of undecidable enumerations (Gödel enumerations of partial recursive functions, for instance). So, if we consider an arbitrary general recursive function f, it may happen that we do not know whether f induces a morphism of suitable enumerated sets. Nevertheless, we have an algorithm on codes of elements of the considered sets. Does there exist "something computable" defined on elements of these sets which corresponds in a natural way to that algorithm? The concept of multimorphism over enumerated sets, introduced and discussed in the paper, is suggested as a possible answer to this question.

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