Notre Dame Journal of Formal Logic
Volume 34, Number 4, Fall 1993

The Fibrational Formulation of Intuitionistic Predicate Logic I: Completeness According to Gödel, Kripke, and Läuchli. Part 2

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Abstract This is the second, concluding part of a two-part paper. After the mainly preliminary work of the first part, the present second part contains the treatment of the fibrational versions of the Kripke and the Läuchli completeness theorems.

The Introduction to the first part (Makkai [4]) covers the present second part as well. The numbering of the sections continues that of the first part.

4 Free objects Free objects will appear in the sequel on three levels. We will use free objects of the category $\mathrm{car}(\boldsymbol{B}, \mathbf{Set})$ in the formulation of the "canonical" Kripke completeness theorem for a general Heyting$^{(-)}$ fibration. We will need free cartesian categories as base categories for the fibrations in our formulation of Läuchli's completeness theorem. Finally, in the same result, the h^{-}-fibration itself has to be free over its base category, in an appropriate sense. In this section, I am going to explain all these various notions of freeness, and I will give a few elementary results concerning them. The contents of this section are entirely elementary.

Let $\boldsymbol{B}$ be a small cartesian category, $L \in \mathrm{car}(\boldsymbol{B}, \mathbf{Set})$. Given any set X in the form of a disjoint union $X = \dot{\bigcup}_{A \in \boldsymbol{B}} X_A$, indexed by the objects of $\boldsymbol{B}$, and a mapping $\varphi : X \to |L| = \dot{\bigcup}_{A \in \boldsymbol{B}} L(A)$ such that $\varphi(X_A) \subset L(A)$ for all $A \in \boldsymbol{B}$ (such a map is called *proper*), we say that L is *free on X via φ* if for any $K \in \mathrm{car}(\boldsymbol{B}, \mathbf{Set})$ and any proper $\psi : X \to |K|$, there is a unique arrow $\ell : L \to K$ in $\mathrm{car}(\boldsymbol{B}, \mathbf{Set})$ such that $\psi = h \circ \varphi$ where the composite $h \circ \varphi$ is defined in the natural way: $(h \circ \varphi)(x) = h_A(\varphi(x))$ for $x \in X_A$. We say that $L \in \mathrm{car}(\boldsymbol{B}, \mathbf{Set})$ is *free* if it is free on some set X.

Received August 8, 1989; revised February 28, 1991