Notre Dame Journal of Formal Logic
Volume 38, Number 2, Spring 1997

# Book Review

Shaughan Lavine. *Understanding the Infinite*. Harvard University Press, Cambridge, 1994, ix + 372 pages.

This ambitious book is shaped around Mycielski's "finite mathematics" and Lavine's way of placing it in the foundations of mathematics. Lavine sees this formal theory of the indefinitely large (yet always with finite models) "as a codification of the actual historical and psychological source of our intuitions concerning the infinite" (p. 9). Thus he is drawn all the way to the historical roots of set theory. Further, he argues that "extrapolation from the theory of indefinitely large size provide[s] rational justification for believing our current theory of [infinite] sets" (p. 249). To support this he goes into Primitive Recursive Arithmetic and schematic versus second-order axiomatization. Finally he offers finite mathematics as a strategy to settle open questions in set theory such as the continuum hypothesis: "Anything that helps to clarify the sources of our axioms may help to suggest more axioms or help to adjudicate between the additional ones that have already been proposed" (p. 9). He says "It is not at all clear" how to apply the method beyond the currently standard axioms and "That is, I claim, partially why we are genuinely unclear about various questions of set theory" (p. 313). But he offers some possibilities which "are, I believe, the sorts of considerations that will be relevant to arguing for new axioms of set theory" (p. 314).

With all this in place, Lavine argues, "the apparently mysterious character of knowledge of the infinite is dissolved" (p. 10). Yet the particulars of that knowledge remain elusive. Lavine says, citing a similar view by Fraenkel, "Our understanding of the foundations of set theory is not much better than d'Alembert's understanding of the foundations of analysis was in the latter half of the eighteenth century" (p. 153).

The scope of the argument is thus extremely broad, and Lavine offers it as a general introduction to the philosophy of set theory. He discusses Gödel's, Quine's, and Putnam's views, and argues extensively with recent work of Kitcher, Maddy, Parsons, Shapiro, and others. He says "The reader who is innocent of mathematical knowledge beyond that taught in high school should be able to read . . . enough for all the major ideas to be presented," and "A reader who learned freshman calculus once, but perhaps does not remember it very well, and who has had a logic course that included

*Received December 29, 1995*