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Book Review

V. V. Rybakov. *Admissibility of Inference Rules*. Elsevier Science, Amsterdam, 1997. 617 pages.

A rule is admissible in a logic L if it can be added without increasing the set of tautologies of L. For example, the rule $\varphi/\forall x.\varphi$ is admissible in predicate logic, since if φ is a theorem, so is $\forall x.\varphi$. The notion of an admissible rule is quite central to logic, but it hardly attracts any attention outside a small group of people. Modern textbooks do not teach a student about consequence relations let alone admissible rules, and it is hard to find other books on logic that do. It is one of the aims of this book to fill this lacuna.

It deals specifically with the question of admissibility of inference rules and here mainly in the context of intermediate and modal logic.[1] Nevertheless, the reader will also learn a good deal about algebraic logic, deductive systems, and modal and intuitionistic logic in general. The book contains six chapters, of which the first two present the general theory of algebraic, modal, and intuitionistic logic, while the remaining four chapters deal with the problem of admissibility of rules in modal and intermediate logic. We shall summarize the first two chapters before entering a review of the book in a more chronological fashion.

Even though the results are more general, we shall often take advantage of the fact that we are dealing with extensions of $\mathsf{K4}$ and superintuitionistic logics. This will eliminate certain complications, into which we will not go since they are not relevant for the main results. We assume that the reader is acquainted at least with modal and intuitionistic logic. For a general introduction we refer to [2]. In what is to follow, we shall try to use the most standard terminology, which is not necessarily the author's own. For example, we shall make use of generalized Kripke-frames rather than models. This will help in the formulation of the results. Recall that a *Kripke-frame* is a pair $\langle F, R\rangle$ where F is a set and $R \subseteq F^2$. A *generalized Kripke-frame* (or *frame* henceforth) is a triple $\mathfrak{F} = \langle F, R, U\rangle$ where $\langle F, R\rangle$ is a Kripke-frame and $U \subseteq \wp(F)$ is closed under intersection, complement, and the operation $\tau(A) := \{y : \text{ if } y\ R\ x \text{ then } x \in A\}$. A Kripke-frame $\langle F, R\rangle$ is often tacitly identified with the general frame $\langle F, R, \wp(F)\rangle$. A *modal algebra* is a quintuple $\mathfrak{A} := \langle A, 1, -, \cap, \tau\rangle$ where $\langle A, 1, -, \cap\rangle$ is a Boolean algebra with unit, complement, and intersection, and

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