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# CONTROLLING FORMAL FIBERS OF PRINCIPAL PRIME IDEALS

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ABSTRACT. Let $(T, \mathfrak{m})$ be a complete local (Noetherian) ring, $S_0$ the prime subring of $T$ and $p \neq 0$ a regular and prime element of $T$. Given a finite set of incomparable prime ideals $C = \{Q_1, \dots, Q_n\}$ of $T$ such that either $Q_i \cap S_0 = (0)$ for all $i$ or $Q_i \cap S_0 = pS_0$ for all $i$, we provide necessary and sufficient conditions for $T$ to be the completion of a local domain $A$ such that $p \in A$ and the formal fiber of $pA$ is semi-local with maximal ideals the elements of $C$. We also show that in a special case the domain $A$ we construct is excellent.

**1. Introduction.** Much research has been devoted to understanding the relationship between a local (Noetherian) ring and its completion. In these efforts, an important question to address has been the following: given a complete local ring $T$ with maximal ideal $\mathfrak{m}$, when is it the completion of a ring $A$ with certain properties? One major result of this type comes from Lech. Specifically, in [**5**], Lech shows that a complete local (Noetherian) ring $T$ is the completion of a local (Noetherian) domain if and only if the following conditions hold.

(1) The prime ring of $T$ is a domain that acts on $T$ without torsion;

(2) Unless equal to (0), the maximal ideal of $T$ does not belong to (0) as an associated prime ideal.

To understand the relationship between a local ring and its completion it is often useful to consider the formal fibers of the ring. If $A$ is a local (Noetherian) ring with maximal ideal $\mathfrak{m}$ and $P$ a prime ideal of $A$, we define the *formal fiber* of $A$ at $P$ to be $\operatorname{Spec}(\widehat{A} \otimes_A k(P))$ where $\widehat{A}$ is the $\mathfrak{m}$-adic completion of $A$ and $k(P) = A_P/PA_P$. It is important to note that there is a one-to-one correspondence between the formal fiber of $A$ at $P$ and the inverse image of $P$ under the map $\operatorname{Spec}\widehat{A} \to \operatorname{Spec} A$. If $A$ is an integral domain, we call the formal fiber

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