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DERIVATIONS OF A RESTRICTED WEYL TYPE ALGEBRA I

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ABSTRACT. Several authors find all the derivations of an algebra [**1, 3, 6**]. For two (non)associative algebras A_1 and A_2; if the additive group $\mathrm{Der}\,(A_1)$ of all the derivations of A_1 and the additive group $\mathrm{Der}\,(A_2)$ of all the derivations of A_2 are not isomorphic, then the two (non)associative algebras are nonisomorphic as algebras [**5**]. A Weyl type nonassociative algebra and its sub-algebra are defined in the papers [**2, 3, 9**]. We find all the derivations of the nonassociative algebra $\overline{WN_{0,0,s_1}}$ in this paper [**4**].

1. Introduction. Generally, there is an infinite dimensional simple algebra with an outer derivation. Thus, it is an interesting problem to find all the derivations of an infinite dimensional (non)associative algebra [**2**]. The Weyl type nonassociative algebras are defined in the papers [**3, 11**]. All the derivations of the restricted Weyl type nonassociative algebras $\overline{WN_{0,0,1_1}}$ and $\overline{WN_{0,0,2_1}}$ are found in the papers [**1, 2**]. In this paper, we find all the derivations of the restricted Weyl type nonassociative algebras $\overline{WN_{0,0,s_1}}$. We show that $\mathrm{Der}\,(\overline{WN_{0,0,s_1}})$ is (s^2+s)-dimensional. The nonassociative algebra $\overline{WN_{0,0,s_1}}$ contains the matrix ring $M_s(\mathbf{F})$, and we show that $\mathrm{Der}\,(M_s(\mathbf{F}))$ is s^2-dimensional [**2, 4**].

2. Preliminaries. Let $\mathbf{F}$ be a field of characteristic zero (not necessarily algebraically closed). Throughout this paper, $\mathbf{N}$ and $\mathbf{Z}$ will denote the nonnegative integers and the integers, respectively. Let $\mathbf{F}[x_1, \dots, x_{m+s}]$ be the polynomial ring with the variables $x_1, \dots, x_{m+s}$. Let $g_1, \dots, g_n$ be given polynomials in $\mathbf{F}[x_1, \dots, x_{m+s}]$. For $n, m, s \in \mathbf{N}$, let us define the commutative, associative $\mathbf{F}$-algebra $F_{g_n,m,s} = \mathbf{F}[e^{\pm g_1}, \dots, e^{\pm g_n}, x_1^{\pm 1}, \dots, x_m^{\pm 1}, x_{m+1}, \dots, x_{m+s}]$ in the formal power series ring $\mathbf{F}[[x_1, \dots, x_{m+s}]]$ which is called a stable algebra

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