TWO PARAMETERS FOR
RAMANUJAN'S THETA-FUNCTIONS
AND THEIR EXPLICIT VALUES

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ABSTRACT. We define two parameters $g_{k,n}$ and $g'_{k,n}$ involving Ramanujan's theta-functions $\psi(q)$ for any positive real numbers k and n. We study several properties of these parameters and find some explicit values of $\psi(q)$ and quotients of $\psi(q)$ and of $\phi(q)$. This work is a sequel to some recent works by J. Yi.

1. Introduction. For $q := e^{2\pi i z}$, $\text{Im}(z) > 0$, define $\psi(q)$ as

$$\psi(q) := \sum_{n=0}^{\infty} q^{n(n+1)/2} = 2^{-1} q^{-1/8} \vartheta_2(0, z),$$

where ϑ_2 is one of the classical theta-functions [15, page 464]. For $q := e^{2\pi i z}$ and $\text{Im}(z) > 0$, we also define

$$\phi(q) := \sum_{n=-\infty}^{\infty} q^{n^2} = \vartheta_3(0, 2z)$$

and

$$f(-q) := (q; q)_{\infty} = q^{-1/24} \eta(z),$$

where ϑ_3 is another classical theta-function [15, page 464] and η denotes the Dedekind eta-function and $(a; q)_{\infty}$ is defined by

$$(a; q)_{\infty} := \prod_{k=0}^{\infty} (1 - aq^k).$$