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REGULARITY OF POINTS IN THE SPECTRUM OF A C^*-ALGEBRA

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ABSTRACT. The relationship between different notions of regularity for the points in the spectrum of a C^*-algebra is investigated. Under certain conditions on the points of the spectrum, the Fell regularity implies Glimm regularity and vice versa. A localized version of the Fell-Dixmier theorem on continuous trace of a C^*-algebra is described.

1. Introduction. Let A be a C^*-algebra, and let $\widehat{A}$ be the spectrum of A, the space of all (equivalence classes of) irreducible representations of A. In [**4**, 4.5.3(iii)] and [**5**, Remark to Theorem 6] two notions of regularity of points in the spectrum $\widehat{A}$ are described. A point $\pi \in \widehat{A}$ is said to be Fell-regular (or a Fell-point) if there exists an $a \in A^+$ (the set of positive elements of A) and a neighborhood V of π such that $\sigma(a)$ is a rank-one projection for all $\sigma \in V$. On the other hand, a point $\pi \in \widehat{A}$ is said to be Glimm-regular if, whenever (e, U) is a pair such that $e \in A$ and U is a neighborhood of π and (i) $\sigma(e)$ is a projection for all $\sigma \in U$, (ii) $\pi(e)$ is a rank-one projection, then there exists a neighborhood U_0 of π with $U_0 \subseteq U$ such that $\sigma(e)$ is rank-one for all $\sigma \in U_0$. A point π is said to be a separated point of $\widehat{A}$ if for each $\sigma \in \widehat{A} \setminus \overline{\{\pi\}}$ there exist disjoint open sets U_1 and U_2 such that $\pi \in U_1$ and $\sigma \in U_2$.

It is known [**5, 6**] that the notions agree if A is liminal with $\widehat{A}$ Hausdorff. We investigate the relation between these notions for more general C^*-algebras. Of course, if $\pi \in \widehat{A}$ is Fell-regular, then, since $\pi(A)$ contains nonzero elements of the algebra of compact operators $K(H_\pi)$, we have $\pi(A) \supseteq K(H_\pi)$ [**4**, 4.1.10]. On the other hand, if $\pi(A) \cap K(H_\pi) = \{0\}$, then, although π cannot be Fell-regular, it is automatically Glimm-regular (by vacuous satisfaction). However, we prove that, if $\pi \in \widehat{A}$ is a separated point, then π is Fell-regular if and only if $\pi(A) \supseteq K(H_\pi)$ and π is Glimm-regular. We give examples to show that if π is not a separated point then (even if $\pi(A)$ contains the

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